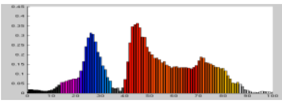


Dr. Qadri Hamarsheh

Signals and Systems

Lecture 4

Outline

- **Manipulation of discrete time signals:**
  - ✓ **Manipulations involving the independent variable n:**
    - ☒ **Shifted in time Operations.**
    - ☒ **Folding, reflection or time reversal.**
    - ☒ **Time Scaling.**
  - ✓ **Manipulations involving the signal amplitude (dependent variable).**

Manipulation of discrete time signals

- **Manipulations involving the independent variable n.**
- **Manipulations involving the signal amplitude (dependent variable).**

Manipulation involving the independent variable n (time):

1. Shifted in time Operations:

Given a D-T signal $x[n]$ and a positive integer p, then

- $y[n] = x[n-p]$ is the p-step right shift of $x[n]$ that results in a **delay** of the signal by p units of time (replacing n by $n-p$).
- $y[n] = x[n+p]$ is the p-step left shift of $x[n]$ that results in an **advance** of the signal by p units of time (replacing n by $n+p$).

Examples:

a) $P_3[n-3]$: Three-step right shift of D-T rectangular pulse $P_3[n]$ (see figure 2-8).

b) $P_3[n+3]$: Three-step left shift of D-T rectangular pulse $P_3[n]$ (see figure 2-8).

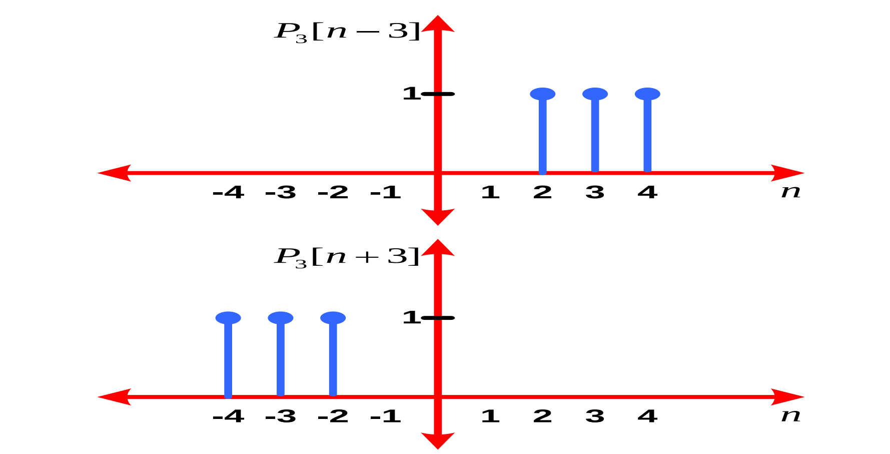
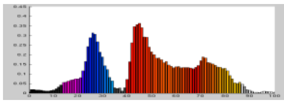


Figure 2_8: Shifted D-T Rectangular Signals

c) The D-T signal

$$x[n] = \begin{cases} 3 & n = 1,2 \\ -2 & n = -1,-2 \\ 0 & n = 0 \text{ and } |n| > 2 \end{cases}$$



Find the time-shifted signal $y[n] = x[n+4]$

Answer:

$$x[n] = \begin{cases} 3 & n = -2,-3 \\ -2 & n = -5,-6 \\ 0 & n = -4\,, n < -6 \quad and\ n > -2 \end{cases}$$

2. Folding, reflection or time reversal:

Let $x[n]$ be the original sequence, and $y[n]$ be reflected sequence, then $y[n]$ is defined by $y[n] = x[-n]$, this means that we replace the independent variable n by $-n$; the result of this operation is a **folding or reflection** of the signal about the time origin $n = 0$.

- It is important to note that the operation of folding and time delaying (or advancing) a signal are **not commutative**:

if SO (shifted operation, for example Time-Delay) and FO (folding operation), we can write:

$SO_k\{x[n]\} = x[n-k]$, k>0.

$FO\{x[n]\}=x[-n]$.

Now:

$SO_k\{FO\{x[n]\}\}= SO_k\{ x[-n]\}= x[-n+k]$ **where as**

$FO\{SO_k\{x[n]\}\}=FO\{x[n-k\}= x[-n-k]$

so :

$$SO_k\{FO\{x[n]\}\} \neq FO\{SO_k\{x[n]\}\}$$

Examples:

a) Show the graphical representation of the signals $x[-n]$ and $x[-n+1]$, where $x[n]$ is the signal illustrated in figure 2-9.

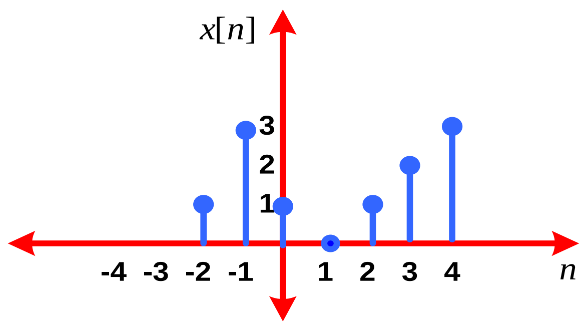


Figure 2_9

Answer: (see figure 2-10)

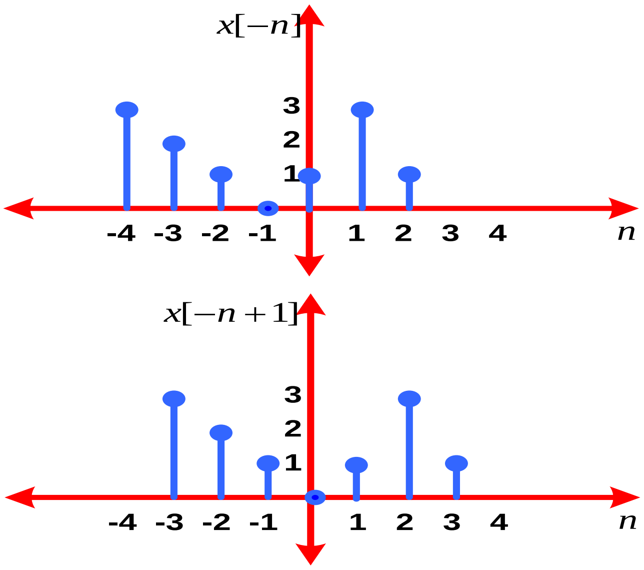


Figure 2_10

A simple way to verify that the result is correct is to compute samples, such as:

$$y[-3] = x[4] = 3,$$
$$y[-2] = x[3] = 2,$$
$$y[-1] = x[2] = 1,$$
$$y[0] = x[1] = 0,$$
$$y[1] = x[0] = 1,$$
$$y[2] = x[-1] = 3,$$
$$y[3] = x[-2] = 1$$

b) The D-T signal

$$x[n] = \begin{cases} +1 & n = 1 \\ -1 & n = -1 \\ 0 & otherwise \end{cases}$$

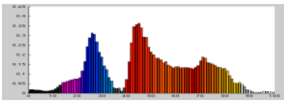
Find the composite signal

$$y[n] = x[n] + x[-n]$$

Answer: $y[n] = 0$; for all integer values of n.

In table 2-1, the precedence rules for the time shifting and the time folding operations are explained.

Table 2-1: Precedence rules for time shifting and time folding	
Order of shifting and folding operations	**Output signal**
1. Folding →Shift to the right	$x[n] \xrightarrow{FO} x[-n] \xrightarrow{SO_R} x[-(n-p)] = x[-n+p]$
2. Shift to the left →Folding	$x[n] \xrightarrow{SO_L} x[n+p] \xrightarrow{FO} x[-n+p]$
3. Folding →Shift to the left	$x[n] \xrightarrow{FO} x[-n] \xrightarrow{SO_L} x[-(n+p)] = x[-n-p]$
4. Shift to the right→Folding	$x[n] \xrightarrow{SO_R} x[n-p] \xrightarrow{FO} x[-n-p]$



3. Time Scaling

Let $x[n]$ denote a D-T signal, then the signal $y[n]$ obtained by scaling the independent variable, time n, by a factor a is defined by

$$y[n] = x[an], a > 0.$$

- ✓ If $a > 1$, the signal is a compressed version of $x[n]$ and some values of the discrete time signal $y[n]$ are lost.
- ✓ if $0 < a < 1$, then the signal $y[n]$ is an expanded version of $x[n]$.

Example:

a) For $a = 2$; in $x[2n]$, the samples $x[n]$ for $n = \pm 1, \pm 3, \pm 5, ...$ are lost.

b) $x[n] = \begin{cases} n & \textit{for n odd} \\ 0 & \textit{otherwise} \end{cases}$

Determine $y[n] = x[2n]$

Answer:

$$y[n] = 0 \quad \textit{for all} \quad n$$

Precedence Rules for time shifting and time scaling:

❖ **C-T case:**

Suppose that $y(t) = x(ct - s)$, this relation between $y(t)$ and $x(t)$ satisfies the conditions:

$$y(0) = x(-s); \text{ and } y(\frac{s}{c}) = x(0)$$

which provide useful checks on $y(t)$ in terms of corresponding values of $x(t)$.

The correct order for time shifting and scaling operations:

a) The **time shifting operation is performed first** on $x(t)$, we get an intermediate signal $v(t) = x(t - s)$; the time shift has replaced t by $t - s$.

b) The **time scaling operation is performed on** $v(t)$, replacing t by ct and the result

$$y(t) = v(ct) = x(ct - s).$$

Examples:

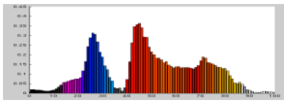
1. Voice signal recorded on a tape recorder:

- ☒ **Compression**: if the tape is played back at a rate faster than the original recording rate.
- ☒ **Expansion**: if the rate is slower than the original.

2. Consider the rectangular pulse $x(t)$ of unit amplitude and a duration of 4 units, depicted in figure 2-11. Find $y(t) = x(3t - 4)$.

Solution:

$c = 3, \quad s = 4 \Rightarrow y(0) = x(-4) = 0; \; y(\frac{s}{c}) = y(\frac{4}{3}) = x(0) = 1$, the graphical solution is represented in figure 2-12.



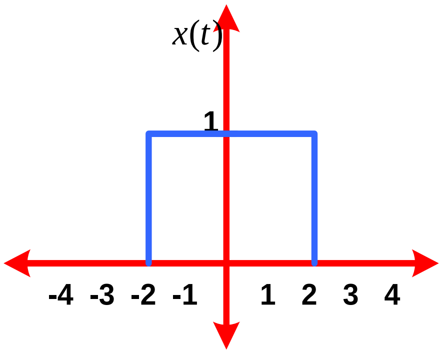


Figure 2_11: Rectangular Signal

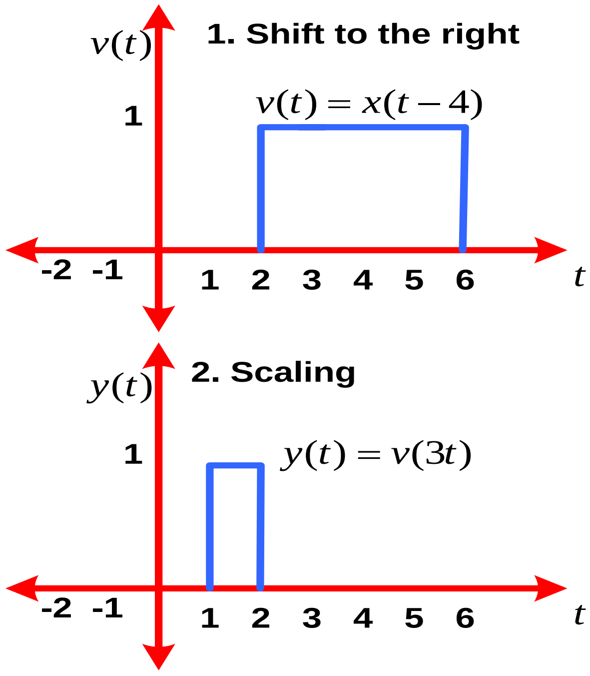


Figure 2_12

❖ D-T case:

The same rules are used in the case of D-T signals, in the following example, these rules are explained.

Example:

Suppose that $x[n] = \{2, -1, \underset{\uparrow}{0}, -3, 4\}$. Find $y[n] = x[3n-4]$

Solution:

$$c = 3, \quad s = 4 \Rightarrow y[0] = x[-4] = 0;\ y[\frac{s}{c}] = y[\frac{4}{3}] = x[0] = 0,$$

The graphical solution is represented in figure 2-13.

To get $y[n] = v[3n]$, we calculate the following points: $y[0], y[1]$ and $y[2]$

$$y[0] = v[0]$$

$$y[1] = v[3]$$

$$y[2] = v[6]$$

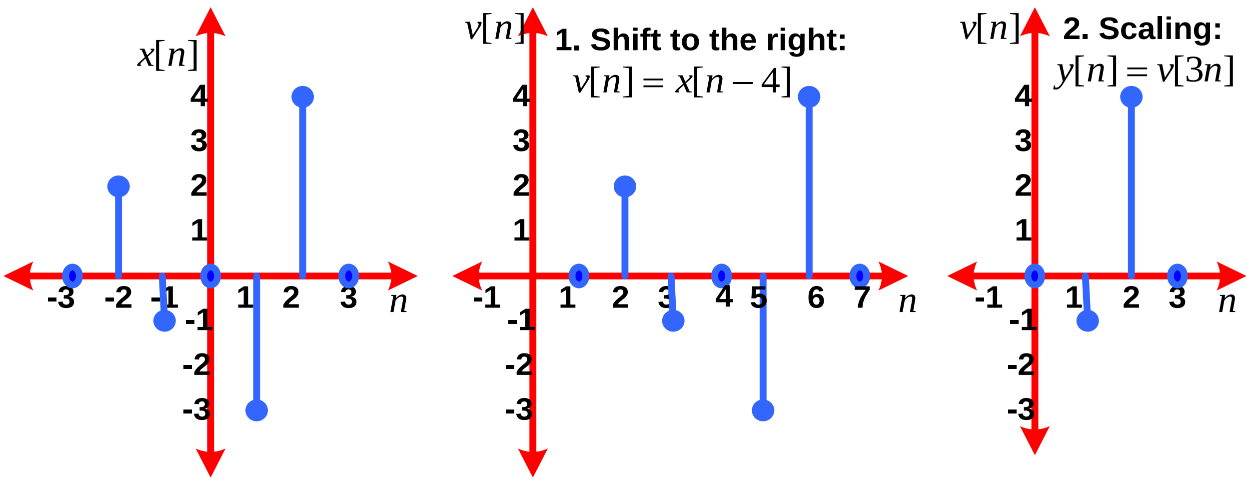
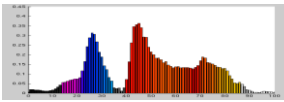


Figure 2_13



Manipulation involving the signal amplitude (dependent variable):

Transformations performed on amplitude (dependent variable) are shown in table 2-2.

Table 2-2: Transformation performed on amplitude

Operation	D-T signals	C-T signals	Physical device
1. Amplitude scaling	$y[n] = cx[n]$	$y(t) = cx(t)$	Electronic amplifier
	c - scaling factor		
2. Addition	$y[n] = x_1[n] + x_2[n]$	$y(t) = x_1(t) + x_2(t)$	Audio mixer
3. Multiplication	$y[n] = x_1[n] \cdot x_2[n]$	$y(t) = x_1(t) \cdot x_2(t)$	Modulator
4. Differentiation	Difference equation	$y(t) = d\frac{x(t)}{dt}$	Inductor
5. Integration	Summation	$y(t) = \int_{-\infty}^{t} x(\tau)d\tau$	Capacitor